
【補足】 テーラ展開による差分近似の例

テーラ展開による差分近似の例について、若干詳しく説明します。

【例題 1】 $\frac{d^2x}{dt^2} = -\omega^2 x$ 以下 $\frac{d^n x}{dt^n} = x^{(n)}$ と表記すると、

$$x^{(2)} = -\omega^2 x, \quad x^{(3)} = -\omega^2 x^{(1)}, \quad x^{(4)} = -\omega^2 x^{(2)} = -\omega^2(-\omega^2 x) = \omega^4 x,$$

$$x^{(5)} = \omega^4 x^{(1)}, \quad x^{(6)} = \omega^4 x^{(2)} = \omega^4(-\omega^2 x) = -\omega^6 x, \quad x^{(7)} = -\omega^6 x^{(1)}$$

$$x^{(8)} = -\omega^6 x^{(2)} = -\omega^6(-\omega^2 x) = \omega^8 x$$

テーラ展開の式

$$x(t+h) = x(t) + h \cdot x^{(1)}(t) + \frac{h^2}{2!} x^{(2)} + \frac{h^3}{3!} x^{(3)} + \frac{h^4}{4!} x^{(4)} + \frac{h^5}{5!} x^{(5)} + \frac{h^6}{6!} x^{(6)} + \frac{h^7}{7!} x^{(7)} + \dots$$

に代入すると、

$$\begin{aligned} x(t+h) &= x(t) + h \cdot x^{(1)}(t) - \frac{h^2 \omega^2}{2!} x(t) - \frac{h^3 \omega^2}{3!} x^{(1)}(t) + \frac{h^4 \omega^4}{4!} x(t) + \frac{h^5 \omega^4}{5!} x^{(1)}(t) \\ &\quad - \frac{h^6 \omega^6}{6!} x(t) - \frac{h^7 \omega^6}{7!} x^{(1)}(t) + \dots \end{aligned}$$

$$\begin{aligned} &= x(t) \left(1 - \frac{h^2 \omega^2}{2!} + \frac{h^4 \omega^4}{4!} - \frac{h^6 \omega^6}{6!} + \dots \right) \\ &\quad + \frac{x^{(1)}(t)}{\omega} \left(h\omega - \frac{h^3 \omega^3}{3!} + \frac{h^5 \omega^5}{5!} x^{(1)}(t) - \frac{h^7 \omega^7}{7!} + \dots \right) = x(t) \cos(h\omega) + \frac{x^{(1)}(t) \sin(h\omega)}{\omega} \end{aligned}$$

一方、

$$x^{(1)}(t+h) = x^{(1)}(t) + h \cdot x^{(2)}(t) + \frac{h^2}{2!} x^{(3)} + \frac{h^3}{3!} x^{(4)} + \frac{h^4}{4!} x^{(5)} + \frac{h^5}{5!} x^{(6)} + \frac{h^6}{6!} x^{(7)} + \frac{h^7}{7!} x^{(8)} + \dots$$

に代入すると

$$\begin{aligned} x^{(1)}(t+h) &= x^{(1)}(t) - h\omega^2 x(t) - \frac{h^2 \omega^2}{2!} x^{(1)}(t) + \frac{h^3 \omega^4}{3!} x(t) + \frac{h^4 \omega^4}{4!} x^{(1)}(t) - \frac{h^5 \omega^6}{5!} x(t) \\ &\quad - \frac{h^6 \omega^6}{6!} x^{(1)}(t) - \frac{h^7 \omega^8}{7!} x(t) + \dots \end{aligned}$$

$$\begin{aligned} &= x^{(1)}(t) \left(1 - \frac{h^2 \omega^2}{2!} + \frac{h^4 \omega^4}{4!} - \frac{h^6 \omega^6}{6!} + \dots \right) \\ &\quad - \omega \cdot x(t) \left(h\omega - \frac{h^3 \omega^3}{3!} + \frac{h^5 \omega^5}{5!} x^{(1)}(t) - \frac{h^7 \omega^7}{7!} + \dots \right) = x^{(1)}(t) \cos(h\omega) - \omega \cdot x(t) \sin(h\omega) \end{aligned}$$

2 【補足】 テーラ展開による差分近似の例

すなわち、以下のように差分近似できる。

$$x(t+h) = x(t)\cos(h\omega) + \frac{x^{(1)}(t)}{\omega}\sin(h\omega)$$

$$x^{(1)}(t+h) = x^{(1)}(t)\cos(h\omega) - \omega \cdot x(t)\sin(h\omega)$$

【Microsoft Excel による式定義】

Excel で以下のように式定義してみよう。

1	A	B	C	D	E	F	G	H	I
	実行パラメータ	値	t	x	dx/dt	x厳密解	dx/dt厳密解	x誤差	dx/dt誤差
2	h	0.1	0	4	0	=4*COS(C2*B\$3)	=-8*SIN(C2*B\$3)	=D2-F2	=E2-G2
3	ω	2	=SUM(B\$2+C2)	=D2*B\$5+E2*B\$7	=E2*B\$5-D2*B\$8	=4*COS(C3*B\$3)	=-8*SIN(C3*B\$3)	=D3-F3	=E3-G3
4	hω	=B2*B3	=SUM(B\$2+C3)	=D3*B\$5+E3*B\$7	=E3*B\$5-D3*B\$8	=4*COS(C4*B\$3)	=-8*SIN(C4*B\$3)	=D4-F4	=E4-G4
5	cos(hω)	=COS(B4)	=SUM(B\$2+C4)	=D4*B\$5+E4*B\$7	=E4*B\$5-D4*B\$8	=4*COS(C5*B\$3)	=-8*SIN(C5*B\$3)	=D5-F5	=E5-G5
6	sin(hω)	=SIN(B4)	=SUM(B\$2+C5)	=D5*B\$5+E5*B\$7	=E5*B\$5-D5*B\$8	=4*COS(C6*B\$3)	=-8*SIN(C6*B\$3)	=D6-F6	=E6-G6
7	sin(hω)/ω	=B6/B3	=SUM(B\$2+C6)	=D6*B\$5+E6*B\$7	=E6*B\$5-D6*B\$8	=4*COS(C7*B\$3)	=-8*SIN(C7*B\$3)	=D7-F7	=E7-G7
8	ω・sin(hω)	=B6*B3	=SUM(B\$2+C7)	=D7*B\$5+E7*B\$7	=E7*B\$5-D7*B\$8	=4*COS(C8*B\$3)	=-8*SIN(C8*B\$3)	=D8-F8	=E8-G8
9			=SUM(B\$2+C8)	=D8*B\$5+E8*B\$7	=E8*B\$5-D8*B\$8	=4*COS(C9*B\$3)	=-8*SIN(C9*B\$3)	=D9-F9	=E9-G9
10			=SUM(B\$2+C9)	=D9*B\$5+E9*B\$7	=E9*B\$5-D9*B\$8	=4*COS(C10*B\$3)	=-8*SIN(C10*B\$3)	=D10-F10	=E10-G10

すると、以下のように、ほぼ厳密解に近い結果が得られる。

	A	B	C	D	E	F	G	H	I
1	実行パラメータ	値	t	x	dx/dt	x厳密解	dx/dt厳密解	x誤差	dx/dt誤差
2	h	0.1	0.0	4.00000	3.92027	4.00000	0.00000	4.00000	3.92027
3	ω	2	0.1	4.30968	2.25277	3.92027	-1.58935	0.38942	3.84212
4	hω	0.2	0.2	4.44756	0.49546	3.68424	-3.11535	0.76331	3.61080
5	cos(hω)	0.980067	0.3	4.40812	-1.28160	3.30134	-4.51714	1.10677	3.23554
6	sin(hω)	0.198669	0.4	4.19294	-3.00757	2.78683	-5.73885	1.40611	2.73128
7	sin(hω)/ω	0.099335	0.5	3.81060	-4.61364	2.16121	-6.73177	1.64940	2.11813
8	ω・sin(hω)	0.397339	0.6	3.27635	-6.03577	1.44943	-7.45631	1.82692	1.42054
9			0.7	2.61148	-7.21728	0.67987	-7.88360	1.93161	0.66632
10			0.8	1.84250	-8.11106	-0.11680	-7.99659	1.95930	-0.11447
11			0.9	1.00006	-8.68147	-0.90881	-7.79078	1.90887	-0.89069
12			1.0	0.11776	-8.90579	-1.66459	-7.27438	1.78234	-1.63141
13			1.1	-0.76924	-8.77505	-2.35400	-6.46797	1.58476	-2.30708
14			1.2	-1.62558	-8.29449	-2.94957	-5.40371	1.32400	-2.89078
15			1.3	-2.41710	-7.48324	-3.42756	-4.12401	1.01045	-3.35923
16			1.4	-3.11227	-6.37367	-3.76889	-2.67991	0.65662	-3.69376
17			1.5	-3.68336	-5.00999	-3.95997	-1.12896	0.27661	-3.88103
18			1.6	-4.10760	-3.44659	-3.99318	0.46699	-0.11442	-3.91358
19			1.7	-4.36809	-1.74578	-3.86719	2.04433	-0.50089	-3.79011
20			1.8	-4.45443	0.02463	-3.58703	3.54016	-0.86740	-3.51553
21			1.9	-4.36319	1.79406	-3.16387	4.89486	-1.19932	-3.10080
22			2.0	-4.09801	3.49196	-2.61457	6.05442	-1.48343	-2.56246
23			2.1	-3.66945	5.05065	-1.96104	6.97261	-1.70840	-1.92195
24			2.2	-3.09460	6.40799	-1.22933	7.61282	-1.86527	-1.20483
25			2.3	-2.39638	7.50986	-0.44861	7.94953	-1.94777	-0.43967
26			2.4	-1.60262	8.31234	0.35000	7.96932	-1.95262	0.34302
27			2.5	-0.74497	8.78343	1.13465	7.67139	-1.87962	1.11203
28			2.6	0.14238	8.90435	1.87407	7.06764	-1.73169	1.83671
29			2.7	1.02405	8.67028	2.53877	6.18212	-1.51472	2.48817
30			2.8	1.86490	8.09056	3.10226	5.05013	-1.23737	3.04042
31			2.9	2.63140	7.18829	3.54208	3.71682	-0.91068	3.47147
32			3.0	3.29299	5.99945	3.84068	2.23532	-0.54769	3.76412

【例題 2】 $\frac{d^2x}{dt^2} = -\mu \frac{dx}{dt}$ 以下 $\frac{d^n x}{dt^n} = x^{(n)}$ と表記すると,

$$\begin{aligned} x^{(2)} &= -\mu x^{(1)}, & x^{(3)} &= -\mu x^{(2)} = -\mu(-\mu x^{(1)}) = \mu^2 x^{(1)}, \\ x^{(4)} &= \mu^2 x^{(2)} = \mu^2(-\mu x^{(1)}) = -\mu^3 x^{(1)}, & x^{(5)} &= -\mu^3 x^{(2)} = -\mu^3(-\mu x^{(1)}) = \mu^4 x^{(1)}, \\ x^{(6)} &= \mu^4 x^{(2)} = \mu^4(-\mu x^{(1)}) = -\mu^5 x^{(1)}, & x^{(7)} &= -\mu^5 x^{(2)} = -\mu^5(-\mu x^{(1)}) = \mu^6 x^{(1)} \end{aligned}$$

テーラ展開の式

$$x^{(1)}(t+h) = x^{(1)}(t) + h \cdot x^{(2)}(t) + \frac{h^2}{2!} x^{(3)}(t) + \frac{h^3}{3!} x^{(4)}(t) + \frac{h^4}{4!} x^{(5)}(t) + \frac{h^5}{5!} x^{(6)}(t) + \frac{h^6}{6!} x^{(7)}(t) + \frac{h^7}{7!} x^{(8)}(t) + \dots$$

に代入すると,

$$\begin{aligned} x^{(1)}(t+h) &= x^{(1)}(t) - h\mu x^{(1)}(t) + \frac{h^2 \mu^2}{2!} x^{(1)}(t) - \frac{h^3 \mu^3}{3!} x^{(1)}(t) + \frac{h^4 \mu^4}{4!} x^{(1)}(t) - \frac{h^5 \mu^5}{5!} x^{(1)}(t) + \dots \\ &= x^{(1)}(t) \left(1 - h\mu + \frac{h^2 \mu^2}{2!} - \frac{h^3 \mu^3}{3!} + \frac{h^4 \mu^4}{4!} - \frac{h^5 \mu^5}{5!} + \dots \right) = x^{(1)}(t) \exp(-h\mu) \end{aligned}$$

一方,

$$x(t+h) = x(t) + h \cdot x^{(1)}(t) + \frac{h^2}{2!} x^{(2)}(t) + \frac{h^3}{3!} x^{(3)}(t) + \frac{h^4}{4!} x^{(4)}(t) + \frac{h^5}{5!} x^{(5)}(t) + \frac{h^6}{6!} x^{(6)}(t) + \frac{h^7}{7!} x^{(7)}(t) + \dots$$

に代入すると

$$\begin{aligned} x(t+h) &= x(t) + h x^{(1)}(t) - \frac{h^2 \mu}{2!} x^{(1)}(t) + \frac{h^3 \mu^2}{3!} x^{(1)}(t) - \frac{h^4 \mu^3}{4!} x^{(1)}(t) + \frac{h^5 \mu^4}{5!} x^{(1)}(t) + \dots \\ &= x(t) + \frac{x^{(1)}(t)}{\mu} - \frac{x^{(1)}(t)}{\mu} \left(1 - h\mu + \frac{h^2 \mu^2}{2!} - \frac{h^3 \mu^3}{3!} + \frac{h^4 \mu^4}{4!} - \frac{h^5 \mu^5}{5!} + \dots \right) \\ &= x(t) + \frac{x^{(1)}(t)}{\mu} (1 - \exp(-h\mu)) \end{aligned}$$

すなわち, 以下のように差分近似できる。

$$\begin{aligned} x(t+h) &= x(t) + \frac{x^{(1)}(t)}{\mu} (1 - \exp(-h\mu)) \\ x^{(1)}(t+h) &= x^{(1)}(t) \exp(-h\mu) \end{aligned}$$

【Microsoft Excel による式定義】

Excel で以下のように式定義してみよう。

	A	B	C	D	E	F	G	H	I
1	実行パラメータ	値	t	x	dx/dt	x厳密解	dx/dt厳密解	x誤差	dx/dt誤差
2	h	0.1	0	2	-4	=4*EXP(-B3*C2)/B3	=-4*EXP(-C2*B3)	=D2-F2	=E2-G2
3	μ	2	=SUM(B\$2+C2)	=D2+E2*B\$7	=E2*B\$5	=4*EXP(-B3*C3)/B3	=-4*EXP(-C3*B3)	=D3-F3	=E3-G3
4	hμ	=B2*B3	=SUM(B\$2+C3)	=D3+E3*B\$7	=E3*B\$5	=4*EXP(-B3*C4)/B3	=-4*EXP(-C4*B3)	=D4-F4	=E4-G4
5	exp(-hμ)	=EXP(-B4)	=SUM(B\$2+C4)	=D4+E4*B\$7	=E4*B\$5	=4*EXP(-B3*C5)/B3	=-4*EXP(-C5*B3)	=D5-F5	=E5-G5
6	1-exp(-hμ)	=1-B5	=SUM(B\$2+C5)	=D5+E5*B\$7	=E5*B\$5	=4*EXP(-B3*C6)/B3	=-4*EXP(-C6*B3)	=D6-F6	=E6-G6
7	(1-exp(-hμ))/μ	=B6/B3	=SUM(B\$2+C6)	=D6+E6*B\$7	=E6*B\$5	=4*EXP(-B3*C7)/B3	=-4*EXP(-C7*B3)	=D7-F7	=E7-G7
8			=SUM(B\$2+C7)	=D7+E7*B\$7	=E7*B\$5	=4*EXP(-B3*C8)/B3	=-4*EXP(-C8*B3)	=D8-F8	=E8-G8
9			=SUM(B\$2+C8)	=D8+E8*B\$7	=E8*B\$5	=4*EXP(-B3*C9)/B3	=-4*EXP(-C9*B3)	=D9-F9	=E9-G9
10			=SUM(B\$2+C9)	=D9+E9*B\$7	=E9*B\$5	=4*EXP(-B3*C10)/B3	=-4*EXP(-C10*B3)	=D10-F10	=E10-G10
11			=SUM(B\$2+C10)	=D10+E10*B\$7	=E10*B\$5	=4*EXP(-B3*C11)/B3	=-4*EXP(-C11*B3)	=D11-F11	=E11-G11

4 【補足】 テーラ展開による差分近似の例

【例題 1】と同様、ほぼ厳密解に近い結果が得られる。

	A	B	C	D	E	F	G	H	I
1	実行パラメータ	値	t	x	dx/dt	x厳密解	dx/dt厳密解	x誤差	dx/dt誤差
2	h	0.1	0.0	2.00000	-2.36254	2.00000	-4.00000	4.00000	1.63746
3	μ	2	0.1	1.78587	-1.93428	1.63746	-3.27492	0.14841	1.34064
4	hμ	0.2	0.2	1.61056	-1.58366	1.34064	-2.68128	0.26992	1.09762
5	exp(-hμ)	0.818731	0.3	1.46703	-1.29659	1.09762	-2.19525	0.36940	0.89866
6	1 - exp(-hμ)	0.181269	0.4	1.34951	-1.06156	0.89866	-1.79732	0.45085	0.73576
7	(1 - exp(-hμ))/μ	0.090635	0.5	1.25330	-0.86913	0.73576	-1.47152	0.51754	0.60239
8			0.6	1.17452	-0.71158	0.60239	-1.20478	0.57213	0.49319
9			0.7	1.11003	-0.58259	0.49319	-0.98639	0.61683	0.40379
10			0.8	1.05722	-0.47699	0.40379	-0.80759	0.65343	0.33060
11			0.9	1.01399	-0.39052	0.33060	-0.66120	0.68340	0.27067
12			1.0	0.97860	-0.31973	0.27067	-0.54134	0.70793	0.22161
13			1.1	0.94962	-0.26178	0.22161	-0.44321	0.72801	0.18144
14			1.2	0.92589	-0.21432	0.18144	-0.36287	0.74446	0.14855
15			1.3	0.90647	-0.17547	0.14855	-0.29709	0.75792	0.12162
16			1.4	0.89056	-0.14367	0.12162	-0.24324	0.76894	0.09957
17			1.5	0.87754	-0.11762	0.09957	-0.19915	0.77797	0.08152
18			1.6	0.86688	-0.09630	0.08152	-0.16305	0.78536	0.06675
19			1.7	0.85815	-0.07885	0.06675	-0.13349	0.79141	0.05465
20			1.8	0.85101	-0.06455	0.05465	-0.10929	0.79636	0.04474
21			1.9	0.84516	-0.05285	0.04474	-0.08948	0.80042	0.03663
22			2.0	0.84037	-0.04327	0.03663	-0.07326	0.80374	0.02999
23			2.1	0.83644	-0.03543	0.02999	-0.05998	0.80645	0.02455
24			2.2	0.83323	-0.02901	0.02455	-0.04911	0.80868	0.02010
25			2.3	0.83060	-0.02375	0.02010	-0.04021	0.81050	0.01646
26			2.4	0.82845	-0.01944	0.01646	-0.03292	0.81199	0.01348
27			2.5	0.82669	-0.01592	0.01348	-0.02695	0.81321	0.01103
28			2.6	0.82525	-0.01303	0.01103	-0.02207	0.81421	0.00903
29			2.7	0.82407	-0.01067	0.00903	-0.01807	0.81503	0.00740
30			2.8	0.82310	-0.00874	0.00740	-0.01479	0.81570	0.00606
31			2.9	0.82231	-0.00715	0.00606	-0.01211	0.81625	0.00496
32			3.0	0.82166	-0.00586	0.00496	-0.00992	0.81670	0.00406

【例題 3】 $\frac{d^2x}{dt^2} + \mu \frac{dx}{dt} + \omega^2 x = 0$ の場合, $x = e^{-\frac{\mu t}{2}} \cdot f$ とおくと,

$$x^{(1)} = -\frac{\mu}{2} e^{-\frac{\mu t}{2}} f + e^{-\frac{\mu t}{2}} f^{(1)}, \quad x^{(2)} = \frac{\mu^2}{4} e^{-\frac{\mu t}{2}} f - \mu e^{-\frac{\mu t}{2}} f^{(1)} + e^{-\frac{\mu t}{2}} f^{(2)}$$

これを元の式に代入すると,

$$\begin{aligned} & \left(\frac{\mu^2}{4} e^{-\frac{\mu t}{2}} f - \mu e^{-\frac{\mu t}{2}} f^{(1)} + e^{-\frac{\mu t}{2}} f^{(2)} \right) + \mu \left(-\frac{\mu}{2} e^{-\frac{\mu t}{2}} f + e^{-\frac{\mu t}{2}} f^{(1)} \right) + \omega^2 e^{-\frac{\mu t}{2}} f \\ & = e^{-\frac{\mu t}{2}} f^{(2)} - \left(\frac{\mu^2}{4} - \omega^2 \right) e^{-\frac{\mu t}{2}} f = e^{-\frac{\mu t}{2}} f^{(2)} - \left(\frac{\mu^2 - 4\omega^2}{4} \right) e^{-\frac{\mu t}{2}} f = 0 \end{aligned}$$

$e^{-\frac{\mu t}{2}}$ で除すと $f^{(2)} - \left(\frac{\mu^2 - 4\omega^2}{4} \right) f = 0$ となり, f に対しては【例題 1】、【例題 2】と同様となる。

【Microsoft Excel による式定義1】

以下のように式定義する(下記の①が 0 となる時は別に定義すること)。

	A	B	C	D	E	F
1	実行パラメータ	値	t	x	dx/dt	f
2	h	0.1	0	4	0	4
3	ω	2	=SUM(B\$2+C2)	=F3*EXP(B\$5*C3)	=B\$5*D3+EXP(B\$5*C3)*G3	=IF(B\$7>=0,F2+G2*B\$12,F2*B\$13+G2*B\$15)
4	μ	2	=SUM(B\$2+C3)	=F4*EXP(B\$5*C4)	=B\$5*D4+EXP(B\$5*C4)*G4	=IF(B\$7>=0,F3+G3*B\$12,F3*B\$13+G3*B\$15)
5	$-\mu/2$	=B4/2	=SUM(B\$2+C4)	=F5*EXP(B\$5*C5)	=B\$5*D5+EXP(B\$5*C5)*G5	=IF(B\$7>=0,F4+G4*B\$12,F4*B\$13+G4*B\$15)
6	$\mu^2-4\omega^2$	=B4^2-4*B3^2	=SUM(B\$2+C5)	=F6*EXP(B\$5*C6)	=B\$5*D6+EXP(B\$5*C6)*G6	=IF(B\$7>=0,F5+G5*B\$12,F5*B\$13+G5*B\$15)
7	$(\mu^2-4\omega^2)/4$	=B6/4	=SUM(B\$2+C6)	=F7*EXP(B\$5*C7)	=B\$5*D7+EXP(B\$5*C7)*G7	=IF(B\$7>=0,F6+G6*B\$12,F6*B\$13+G6*B\$15)
8	絶対値	=ABS(B7)	=SUM(B\$2+C7)	=F8*EXP(B\$5*C8)	=B\$5*D8+EXP(B\$5*C8)*G8	=IF(B\$7>=0,F7+G7*B\$12,F7*B\$13+G7*B\$15)
9	①平方根	=SQRT(B8)	=SUM(B\$2+C8)	=F9*EXP(B\$5*C9)	=B\$5*D9+EXP(B\$5*C9)*G9	=IF(B\$7>=0,F8+G8*B\$12,F8*B\$13+G8*B\$15)
10	②= $\textcircled{1} \times h$	=B9*B2	=SUM(B\$2+C9)	=F10*EXP(B\$5*C10)	=B\$5*D10+EXP(B\$5*C10)*G10	=IF(B\$7>=0,F9+G9*B\$12,F9*B\$13+G9*B\$15)
11	$\exp(-\textcircled{2})$	=EXP(-B10)	=SUM(B\$2+C10)			
12	$(1-\exp(-\textcircled{2}))/\textcircled{1}$	=B11/B9	=SUM(B\$2+C11)			
13	$\cos(\textcircled{2})$	=COS(B10)	=SUM(B\$2+C12)			
14	$\sin(\textcircled{2})$	=SIN(B10)	=SUM(B\$2+C13)			
15	$\sin(\textcircled{2})/\textcircled{1}$	=B14/B9	=SUM(B\$2+C14)			
16	$\sin(\textcircled{2})*\textcircled{1}$	=B14*B9	=SUM(B\$2+C15)			
17			=SUM(B\$2+C16)			
18			=SUM(B\$2+C17)			

	G	H	I
	dt/dt	x厳密解	x厳密解との差
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C2),4*EXP(B\$5*C2)*COS((B\$9*C2))	=D2-H2
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C3),4*EXP(B\$5*C3)*COS((B\$9*C3))	=D3-H3
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C4),4*EXP(B\$5*C4)*COS((B\$9*C4))	=D4-H4
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C5),4*EXP(B\$5*C5)*COS((B\$9*C5))	=D5-H5
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C6),4*EXP(B\$5*C6)*COS((B\$9*C6))	=D6-H6
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C7),4*EXP(B\$5*C7)*COS((B\$9*C7))	=D7-H7
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C8),4*EXP(B\$5*C8)*COS((B\$9*C8))	=D8-H8
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C9),4*EXP(B\$5*C9)*COS((B\$9*C9))	=D9-H9
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C10),4*EXP(B\$5*C10)*COS((B\$9*C10))	=D10-H10
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C11),4*EXP(B\$5*C11)*COS((B\$9*C11))	=D11-H11
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C12),4*EXP(B\$5*C12)*COS((B\$9*C12))	=D12-H12
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C13),4*EXP(B\$5*C13)*COS((B\$9*C13))	=D13-H13
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C14),4*EXP(B\$5*C14)*COS((B\$9*C14))	=D14-H14
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C15),4*EXP(B\$5*C15)*COS((B\$9*C15))	=D15-H15
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C16),4*EXP(B\$5*C16)*COS((B\$9*C16))	=D16-H16
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C17),4*EXP(B\$5*C17)*COS((B\$9*C17))	=D17-H17
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C18),4*EXP(B\$5*C18)*COS((B\$9*C18))	=D18-H18
		=IF(B\$7>=0.4*EXP((B\$5-B\$9)*C19),4*EXP(B\$5*C19)*COS((B\$9*C19))	=D19-H19

この結果も厳密解に近い結果が得られる。

	A	B	C	D	E	F	G	H	I
1	実行パラメータ	値	t	x	dx/dt	f	dt/dt	x厳密解	x厳密解との差
2	h	0.1	0.0	4.00000	-4.00000	4.00000	0.00000	4.00000	0.00000
3	ω	2	0.1	3.56520	-4.64558	3.94015	-1.19401	3.56520	0.00000
4	μ	2	0.2	3.08038	-5.00627	3.76239	-2.35229	3.08038	0.00000
5	$-\mu/2$	-1	0.3	2.57215	-5.12069	3.47204	-3.44017	2.57215	0.00000
6	$\mu^2-4\omega^2$	-12	0.4	2.06310	-5.02935	3.07779	-4.42511	2.06310	0.00000
7	$(\mu^2-4\omega^2)/4$	-3	0.5	1.57179	-4.77283	2.59144	-5.27763	1.57179	0.00000
8	絶対値	3	0.6	1.11273	-4.39035	2.02753	-5.97221	1.11273	0.00000
9	①平方根	1.732051	0.7	0.69669	-3.91857	1.40296	-6.48808	0.69669	0.00000
10	②= $\textcircled{1} \times h$	0.173205	0.8	0.33089	-3.39072	0.73640	-6.80978	0.33089	0.00000
11	$\exp(-\textcircled{2})$	0.840965	0.9	0.01943	-2.83603	0.04780	-6.92771	0.01943	0.00000
12	$(1-\exp(-\textcircled{2}))/\textcircled{1}$	0.485531	1.0	-0.23626	-2.27942	-0.64223	-6.83832	-0.23626	0.00000
13	$\cos(\textcircled{2})$	0.985037	1.1	-0.43707	-1.74134	-1.31303	-6.54430	-0.43707	0.00000
14	$\sin(\textcircled{2})$	0.17234	1.2	-0.58569	-1.23787	-1.94455	-6.05443	-0.58569	0.00000
15	$\sin(\textcircled{2})/\textcircled{1}$	0.099501	1.3	-0.68620	-0.78094	-2.51788	-5.38339	-0.68620	0.00000
16	$\sin(\textcircled{2})*\textcircled{1}$	0.298502	1.4	-0.74370	-0.37862	-3.01585	-4.55125	-0.74370	0.00000
17			1.5	-0.76390	-0.03555	-3.42358	-3.58291	-0.76390	0.00000
18			1.6	-0.75284	0.24662	-3.72886	-2.50736	-0.75284	0.00000
19			1.7	-0.71659	0.46873	-3.92255	-1.35677	-0.71659	0.00000
20			1.8	-0.66101	0.63364	-3.99886	-0.16558	-0.66101	0.00000
21			1.9	-0.59162	0.74576	-3.95550	1.03057	-0.59162	0.00000
22			2.0	-0.51343	0.81061	-3.79377	2.19587	-0.51343	0.00000
23			2.1	-0.43087	0.83442	-3.51852	3.29547	-0.43087	0.00000
24			2.2	-0.34770	0.82376	-3.13797	4.29644	-0.34770	0.00000
25			2.3	-0.26704	0.78526	-2.66352	5.16885	-0.26704	0.00000
26			2.4	-0.19136	0.72538	-2.10936	5.88658	-0.19136	0.00000
27			2.5	-0.12248	0.65013	-1.49208	6.42815	-0.12248	0.00000
28			2.6	-0.06166	0.56504	-0.83015	6.77736	-0.06166	0.00000
29			2.7	-0.00964	0.47495	-0.14338	6.92375	-0.00964	0.00000
30			2.8	0.03330	0.38403	0.54769	6.86295	0.03330	0.00000
31			2.9	0.06726	0.29572	1.22236	6.59678	0.06726	0.00000
32			3.0	0.09263	0.21273	1.86046	6.13320	0.09263	0.00000

6 【補足】 テーラ展開による差分近似の例

【Microsoft Excel による式定義 2】

以下のように式定義する(下記の①が 0 となる時は別に定義すること)。なお、このケースでは結果が指数関数となるため、厳密解と初期微分値を合わせるために df/dt の初期値を調整している。

1	A	B	C	D	E	F	G	H	I
2	実行パラメータ	値	t	x	dx/dt	f	df/dt	x 厳密解	x 厳密解との差
2	h	0.1	0	4	$=B5*D2+EXP(B5*C2)*G2$	4			
3	ω	1	$=SUM(B2+C2)$	$=F3*EXP(B5*C3)$	$=B5*D3+EXP(B5*C3)*G3$	$=IF(B7>0,F2+G2*B12,F2*B13+G2*B15)$			
4	μ	3	$=SUM(B2+C3)$	$=F4*EXP(B5*C4)$	$=B5*D4+EXP(B5*C4)*G4$	$=IF(B7>0,F3+G3*B12,F3*B13+G3*B15)$			
5	$-\mu/2$	$=B4/2$	$=SUM(B2+C4)$	$=F5*EXP(B5*C5)$	$=B5*D5+EXP(B5*C5)*G5$	$=IF(B7>0,F4+G4*B12,F4*B13+G4*B15)$			
6	$\mu^2-4\omega^2$	$=B4^2-4*B3^2$	$=SUM(B2+C5)$	$=F6*EXP(B5*C6)$	$=B5*D6+EXP(B5*C6)*G6$	$=IF(B7>0,F5+G5*B12,F5*B13+G5*B15)$			
7	$(\mu^2-4\omega^2)/4$	$=B6/4$	$=SUM(B2+C6)$	$=F7*EXP(B5*C7)$	$=B5*D7+EXP(B5*C7)*G7$	$=IF(B7>0,F6+G6*B12,F6*B13+G6*B15)$			
8	絶対値	$=ABS(B7)$	$=SUM(B2+C7)$	$=F8*EXP(B5*C8)$	$=B5*D8+EXP(B5*C8)*G8$	$=IF(B7>0,F7+G7*B12,F7*B13+G7*B15)$			
9	①平方根	$=SQRT(B8)$	$=SUM(B2+C8)$	$=F9*EXP(B5*C9)$	$=B5*D9+EXP(B5*C9)*G9$	$=IF(B7>0,F8+G8*B12,F8*B13+G8*B15)$			
10	②= $\textcircled{1} \times h$	$=B9*B2$	$=SUM(B2+C9)$	$=F10*EXP(B5*C10)$	$=B5*D10+EXP(B5*C10)*G10$	$=IF(B7>0,F9+G9*B12,F9*B13+G9*B15)$			
11	$\exp(-\textcircled{2})$	$=EXP(-B10)$	$=SUM(B2+C10)$	$=F11*EXP(B5*C11)$	$=B5*D11+EXP(B5*C11)*G11$	$=IF(B7>0,F10+G10*B12,F10*B13+G10*B15)$			
12	$(1-\exp(-\textcircled{2}))/\textcircled{1}$	$=B11/B9$	$=SUM(B2+C11)$	$=F12*EXP(B5*C12)$	$=B5*D12+EXP(B5*C12)*G12$	$=IF(B7>0,F11+G11*B12,F11*B13+G11*B15)$			
13	$\cos(\textcircled{2})$	$=COS(B10)$	$=SUM(B2+C12)$	$=F13*EXP(B5*C13)$	$=B5*D13+EXP(B5*C13)*G13$	$=IF(B7>0,F12+G12*B12,F12*B13+G12*B15)$			
14	$\sin(\textcircled{2})$	$=SIN(B10)$	$=SUM(B2+C13)$	$=F14*EXP(B5*C14)$	$=B5*D14+EXP(B5*C14)*G14$	$=IF(B7>0,F13+G13*B12,F13*B13+G13*B15)$			
15	$\sin(\textcircled{2})/\textcircled{1}$	$=B14/B9$	$=SUM(B2+C14)$	$=F15*EXP(B5*C15)$	$=B5*D15+EXP(B5*C15)*G15$	$=IF(B7>0,F14+G14*B12,F14*B13+G14*B15)$			
16	$\sin(\textcircled{2})*\textcircled{1}$	$=B14*B9$	$=SUM(B2+C15)$	$=F16*EXP(B5*C16)$	$=B5*D16+EXP(B5*C16)*G16$	$=IF(B7>0,F15+G15*B12,F15*B13+G15*B15)$			
17			$=SUM(B2+C16)$	$=F17*EXP(B5*C17)$	$=B5*D17+EXP(B5*C17)*G17$	$=IF(B7>0,F16+G16*B12,F16*B13+G16*B15)$			
18			$=SUM(B2+C17)$	$=F18*EXP(B5*C18)$	$=B5*D18+EXP(B5*C18)*G18$	$=IF(B7>0,F17+G17*B12,F17*B13+G17*B15)$			

同様に厳密解に近い結果が得られる。

1	A	B	C	D	E	F	G	H	I
2	実行パラメータ	値	t	x	dx/dt	f	df/dt	x 厳密解	x 厳密解との差
2	h	0.1	0.0	4.00000	-6.52902	4.00000	-0.52902	4.00000	0.00000
3	ω	1	0.1	3.07865	-5.02514	3.57688	-0.47306	3.07865	0.00000
4	μ	3	0.2	2.36952	-3.86767	3.19852	-0.42302	2.36952	0.00000
5	$-\mu/2$	-1.5	0.3	1.82373	-2.97680	2.86018	-0.37827	1.82373	0.00000
6	$\mu^2-4\omega^2$	5	0.4	1.40366	-2.29113	2.55764	-0.33826	1.40366	0.00000
7	$(\mu^2-4\omega^2)/4$	1.25	0.5	1.08035	-1.76340	2.28709	-0.30248	1.08034	0.00000
8	絶対値	1.25	0.6	0.83150	-1.35722	2.04516	-0.27048	0.83150	0.00000
9	①平方根	1.118034	0.7	0.63998	-1.04460	1.82883	-0.24187	0.63997	0.00000
10	②= $\textcircled{1} \times h$	0.111803	0.8	0.49257	-0.80399	1.63538	-0.21629	0.49256	0.00000
11	$\exp(-\textcircled{2})$	0.89422	0.9	0.37911	-0.61880	1.46239	-0.19341	0.37911	0.00000
12	$(1-\exp(-\textcircled{2}))/\textcircled{1}$	0.799815	1.0	0.29179	-0.47627	1.30770	-0.17295	0.29178	0.00000
13	$\cos(\textcircled{2})$	0.993757	1.1	0.22458	-0.36657	1.16937	-0.15465	0.22458	0.00000
14	$\sin(\textcircled{2})$	0.111571	1.2	0.17285	-0.28213	1.04568	-0.13829	0.17285	0.00000
15	$\sin(\textcircled{2})/\textcircled{1}$	0.099792	1.3	0.13304	-0.21715	0.93507	-0.12367	0.13303	0.00000
16	$\sin(\textcircled{2})*\textcircled{1}$	0.12474	1.4	0.10239	-0.16713	0.83616	-0.11058	0.10239	0.00000
17			1.5	0.07881	-0.12863	0.74771	-0.09889	0.07881	0.00000
18			1.6	0.06066	-0.09901	0.66862	-0.08843	0.06065	0.00000
19			1.7	0.04668	-0.07620	0.59790	-0.07907	0.04668	0.00000
20			1.8	0.03593	-0.05865	0.53465	-0.07071	0.03593	0.00000
21			1.9	0.02766	-0.04514	0.47810	-0.06323	0.02765	0.00000
22			2.0	0.02129	-0.03474	0.42753	-0.05654	0.02128	0.00000
23			2.1	0.01638	-0.02674	0.38231	-0.05056	0.01638	0.00000
24			2.2	0.01261	-0.02058	0.34187	-0.04521	0.01261	0.00000
25			2.3	0.00970	-0.01584	0.30571	-0.04043	0.00970	0.00000
26			2.4	0.00747	-0.01219	0.27337	-0.03615	0.00747	0.00000
27			2.5	0.00575	-0.00938	0.24446	-0.03233	0.00575	0.00000
28			2.6	0.00442	-0.00722	0.21860	-0.02891	0.00442	0.00000
29			2.7	0.00341	-0.00556	0.19548	-0.02585	0.00341	0.00000
30			2.8	0.00262	-0.00428	0.17480	-0.02312	0.00262	0.00000
31			2.9	0.00202	-0.00329	0.15631	-0.02067	0.00202	0.00000
32			3.0	0.00155	-0.00253	0.13978	-0.01848	0.00155	0.00000